Journal of
INDUSTRIAL
TECHNOLOGY

Volume 17, Number 1 - November 2000 to January 2001

Determining Process Mean for Machining while Unbalanced Tolerance Design Occurs

By Dr. Ming-Hsien Caleb Li & Dr. Joseph C. Chen

KEYWORD SEARCH

Machine Tools
Manufacturing
Production
Quality Control

Reviewed Article

The Official Electronic Publication of the National Association of Industrial Technology • www.nait.org

Determining Process Mean for Machining while Unbalanced Tolerance Design Occurs

By Dr. Ming-Hsien Caleb Li & Dr. Joseph C. Chen

Dr. Ming-Hsien Caleb Li is an Associate Professor in the Department of Industrial Engineering at Feng Chia University, Taiwan. His interests are quality engineering, Taguchi method, experimental design, and statistical quality control. Li is a member of IIE and ASQ.

Dr. Joseph C. Chen, PE is an associate professor in the Department of Industrial Education and Technology at Iowa State University. Recently, he received the Early Achievement in Teaching Award from Iowa State University. His research interests are in cellular manufacturing systems, machine tool dynamics, and intelligent control as it relates to neural networks and fuzzy control.

Introduction

Industrial needs have driven a demand for increased design and drafting skills in the Industrial Technology discipline, especially when it comes to tolerance. It is now widely accepted that over 70 percent of final production costs are incurred during design (Boothroyd, Dewhurst, and Knight, 1994 P. 2). Therefore, it is important for industrial technologists to realize how the design process affects the overall product cost while they are preparing process plans for a particular product. In particular, it is very important to understand the tolerance and identify the process mean based on the tolerance given by the designer. Consequently, the most critical stage of design processes is identifying tolerances. Tight tolerance specifications lead to increased manufacturing costs, but broad tolerance specifications could cause the product assembly to fall short of its functional requirements. Proper tolerance is the key to this movement from design to manufacturing with low cost and high quality.

Tolerance is defined in three ways: bilateral, unilateral, and unbalanced (Liggett, 1993). Bilateral tolerances are most common in industry, but more and more product designs are using unbalanced tolerances, for example, to establish the fit between two parts composed of different materials. In this case, different thermal expansion coefficients lead to more expansion in one direction than in another. This case and others like it have created a need for more robust design practices that consider a variety of input data. Furthermore, research has shown that robust design practices can lead to low-cost improvements in quality, manufacturability, and reliability (Taguchi, 1991). Taguchi's introduction of robust design resulted in significant improvements in the manufacturing processes and product quality of several major American industrial firms. The objective of robust design aims to reduce output variation from the target value by making performance characteristics insensitive to noise factors such as manufacturing imperfections, environmental variations, and deterioration.

Quadratic Loss Function for Quality

Taguchi redefines quality using the quadratic loss function, which is used to calculate the loss that a product imparts to society from the time of shipment. Variables for static quality characteristics are continuous and divided into three types: the-smaller-the-better (STB), the-larger-the-better (LTB), and the-nominal-the-best (NTB). Assuming that y is a quality characteristic, the quality loss functions are defined as follows:

$$L(y) = \begin{cases} ky^2 & \text{for STB} \\ k/y^2 & \text{for LTB} \\ k(y-T)^2 & \text{for NTB} \end{cases} \qquad (1)$$

where k is the coefficient of the quality loss function and T is the target value of a nominal dimension.

Many quality characteristics belong to NTB. For example, a tolerance of 4.650" ± 0.005" means that a part would still be acceptable if the final measurement of the machined part ranged anywhere from 4.645" to 4.655". In this case, the nominal value is 4.650", which is the median number of the range. The nominal value is also a target value for machine setup or CNC programming.

In certain situations, however, deviations in quality characteristics in one direction are much more critical than deviations in the opposite direction. An example of their occurrence is the fit between two parts with different materials: The thermal expansion is different due to different coefficients and it is usually much more harmful to expand in one direction than the other. For example, consider a tolerance of 4.650" + 0.005"/ -0.002", which is an example of an unbalanced tolerance design. In this case, an asymmetric loss function can be introduced as follows (Phakde, 1989):

$$L(y) = \begin{cases} k_1(y-T)^2 & \text{for } y \le T \\ k_2(y-T)^2 & \text{for } y \ge T \end{cases} \qquad (2)$$

where k_1 and k_2 are assigned values based on the critical situation of each direction of tolerance. Then, the target value of the process mean is identified in order to guide the NC programmer or machine operator in minimizing quality loss. The purpose of this study is to answer the following question: Does the nominal value of upper and lower specification limits minimize quality loss, or does their center of average value minimize quality loss?

Problem Statement and Objectives

The purpose of this paper is to determine the target value of the process mean for minimizing quality loss in a situation where an unbalanced tolerance design has been applied in a manufacturing setting. The quality loss function of this unbalanced tolerance design and an analytical approach for determining the process mean will be discussed. These approaches allowed the machine operator or Numerical Control (NC) programmer to set the machine or develop an NC code for the product. With this approach, the authors hope that the goal of minimized quality loss could be approached.

Literature Review

Taguchi et. al (1989) gave several examples of NTB quality characteristics with asymmetrical specification limits, in which they simply set the tolerance at the smaller tolerance limit, i.e. $\Delta = \min(\Delta_1, \Delta_2)$, where Δ_1 represents the tolerance on the lower side and δ_2 represents the tolerance on the higher side. Setting the process mean at the target value could not minimize the expected quality loss, especially when the coefficient of the asymmetrical loss function, k_1, differs greatly from k_2.

Parkinson et. al (1993) described a general, rigorous approach for robust optimal tolerance design. The method allows designers to explicitly consider and control, as an integrated part of the optimization process, the effects of variability in design variables and parameters on a design. Variability is defined in terms of tolerances, which bracket the variation of fluctuating quantities. A designer can apply tolerances to any model input and analyze how the tolerances affect the design using either a worst-case scenario or a statistical analysis. Jeang (1998, 1999) proposed analyzing quality loss and manufacturing costs in one model so that tolerance can be determined simultaneously. The research focused on the optimal range of tolerance, but setting the process mean to minimize expected quality loss was not considered. Wu and Tang (1998) provided a design method for allocating dimensional tolerances with asymmetric quality losses. For adjustment of the process mean, the authors discussed two kinds of distribution: uniform distribution and normal distribution. In normal distribution, the authors used a truncated range rather than an infinitive range to simplify the computation and gave inaccurate process adjustment.

Several studies discussing optimal process mean settings have been reported in the literature. For example, Hunter and Kartha (1977) formulated the optimal process problem with the assumption that underfilled cans could be sold in a secondary market for a fixed price. Bisgaard et al. (1984) suggested that this assumption was unrealistic because it implied that empty and full cans could be sold for the same price. They eliminated this unreasonable assumption by extending Hunter and Karths' model to a situation where the underfilled cans were sold in the secondary market at a reduced price. However, some products, such as pharmaceuticals, may only be sold in the regular market; additionally, a secondary market may be so distant that transportation and other related costs may make selling a substandard product in a secondary market impractical.

Golhar (1987) provided the alternative of emptying and refilling cans despite the risk of an associated reprocessing cost. He formulated such a canning problem and found the optimal process mean setting, but his assumption that overfilled cans, no matter the amount of fill, could be sold for a fixed price was unrealistic. Golhar and Pollock (1988) extended Golhar's model to a process where both the process mean and the upper limit could be controlled to minimize the cost of rejects and of excess raw material. However, these models ignored the effect of process mean setting on inventories, production, and recycling lot sizes. Gupta and Golhar (1991) presented a model that combined production and quality costs to determine the optimal lot sizing parameters and process mean. In this paper, only the lower specification limit was used to decide the quality characteristics of the model. Many manufacturing processes involve quality characteristics with specifications on both sides of the target value. Pfeifer (1999) presented a general piecewise canning problem model. This model was much more flexible and included several previous models as special cases.

This paper discusses target selection of the process mean with asymmetrical quality loss functions. The exact shifting of the process mean is found.

Target Selection of the Process Mean with Quadratic Loss Function

Suppose that the quality characteristic, *y*, is normally distributed with mean μ and variance σ^2.

$$E[L(y)] = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{T} k_1(y-T)^2 \exp\left(-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right) dy + \frac{1}{\sigma\sqrt{2\pi}}\int_{T}^{\infty} k_2(y-T)^2 \exp\left(-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right) dy \qquad (3)$$

The variance σ^2 is independent of the process mean μ. That is, when μ is adjusted by an adjusting factor, variance stays in a state of statistical control if no assignable cause exists. The expected quality loss function, E[L(y)] is shown in equation (3).

If $\delta = (\mu - T)/\sigma$, $z = (y - \mu)/\sigma$,

then $E[L(y)]$ can be expressed as $EL(\delta)$ and the exact relationship among $EL(\delta)$, k_1, k_2 and σ^2 is shown in equation *(4)*.

$$EL(\delta)=(k_2-k_1)\delta\sigma^2\phi(\delta)+k_2\sigma^2(1+\delta^2)+(k_1-k_2)\sigma^2(1+\delta^2)\Phi(-\delta) \quad (4)$$

where $\Phi(z)$ is the cumulative probability function and $\psi(z)$ is the normal probability density function. Therefore, the expected quality loss is a function of k_1, k_2, δ, and σ. Let $EL(\delta_o)$ be the optimal expected quality loss resulting from the process mean at μ_o such that $EL'(\delta_o) = 0$, where $\delta_o = (\mu_o - T)/\sigma$. Thus, $EL(\delta_o)$ is the minimum expected quality loss. Note that $EL(0)$ is the expected quality loss associated with the process mean at the target value T. Assuming that $R_k = k_1/k_2$ and $R_L = EL(0)/EL(\delta_o)$, the relationship among R_k, δ_o and R_L is shown in Table 1. The relationship between δ_o and R_k is shown in Figure 1. The relationship between R_L and R_k is shown in Figure 2.

Analysis of the Results

The factors that influence the optimal value of δ and the relationships among $EL(\delta_o)$, R_k, δ_o, and R_L are summarized as follows.

1. If $k_1 > k_2$ ($k_1 < k_2$), where $k_1 > k_2$ means that the coefficient of loss function on the left hand side is larger than right hand side, in other words, it is more harmful to deviate to the left than to the right ($R_k > 1$ ($R_k < 1$)), then the value of δ_o is positive (negative). That is, the process mean should be set a bit to the right (left) of the target value. The optimal setting of the process mean is: $T + \delta_o\sigma$ ($T - \delta_o\sigma$). The larger values of R_k ($1/R_k$), leads to a larger shift of values to the right (left).

Example 1: the specification of a metal component is 5.080" –0.012"/ +0.015". The quality loss at the specification limits are \$1.44 and \$0.81, respectively, then $k_1=1.44/(-0.012)^2= 10000$ and $k_2=0.818/(0.015)^2=2500$, so $R_k=4$. Table 1 shows that $\delta_o = 0.5392$ and $R_L = 1.3338$. In other words, the process mean should be shifted to the right by 0. 5392σ from

Table 1: The relationship between R_k, δ_o, and R_L for quadratic loss functions

R_k	δ_o	R_L
1.0000	0.0000	1.0000
2.0000	0.2760	1.0782
3.0000	0.4363	1.2029
4.0000	0.5492	1.3338
5.0000	0.6360	1.4638
6.0000	0.7065	1.5912
7.0000	0.7658	1.7157
8.0000	0.8168	1.8374
9.0000	0.8616	1.9565
10.0000	0.9015	2.0731

Figure 1. The relationship between δ_o and R_k.

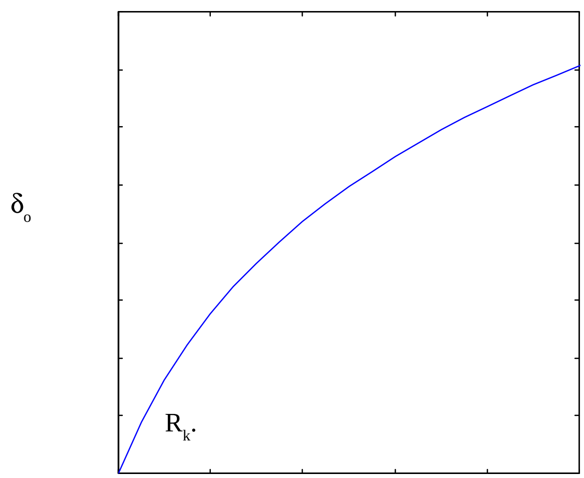
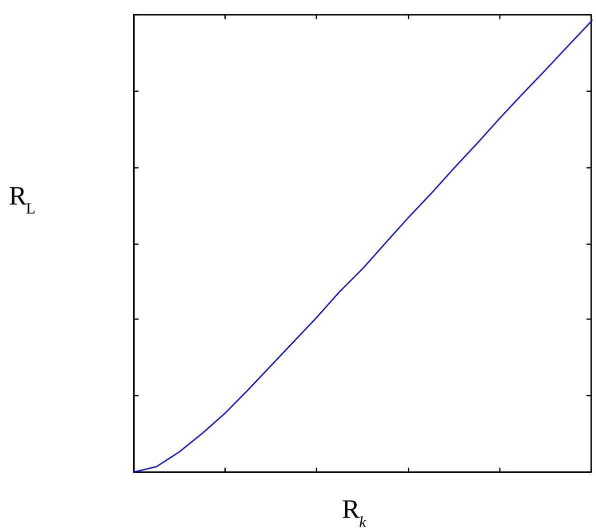


Figure 2. The relationship between R_L (The quality loss ratio) and R_k (The asymmetrical ratio of the coefficient of the quality loss function).



the target value of 5.080". If the standard deviation of this quality characteristic is 0.004, then the optimal value of the process mean, μ_o , is 5.082". The expected quality loss from setting the process mean at a target value is 1.3338 times that of the shifted mean. On the other hand, if $R_k = 1/4$, then $1/R_k = 4$, $\delta_o = -0.5392$, $\mu_o = 5.078"$ and $R_L = 1.3338$.

In an unbalanced tolerance design, since $A_1 = k_1\Delta_1^2$ and $A_2 = k_2\Delta_2^2$, the quality losses on both sides are the same, i.e., $A_1 = A_2$; then the asymmetrical ratio, R_k, can be computed directly from Δ_1 and Δ_2 (i.e., $A_1 = k_1\Delta_1^2 = A_2 = k_2\Delta_2^2$, then $R_k = k_1/k_2 = \Delta_2^2/\Delta_1^2$). Once R_k is determined, the value of δ_o can be obtained from Table 1. Therefore, it is highly recommended that the designer of a product should design the unbalanced tolerances in a way that ensures that the quality losses at the specification limits are equal. Recalling the example above, assume a tolerance band of 0.030". With the same tolerance band, if the unbalanced tolerances change –0.010" and 0.020", instead of –0.012" and +0.018", respectively, then the quality loss for the new specification limits on both sides are equal to $1.0 and the asymmetrical ratio can be obtained directly from the tolerances, i.e. $R_k = (-0.010/0.020)^2 = 4$.

2. If k_1 increases (decreases) proportionally with k_2, i.e. R_k stays fixed, then δ_o and R_L will not change, but from equation (2) and (4), it is clear that EL(0) and EL(δ_o) will increase (decrease) proportionally.

Example 2: if $k_1 = 30000$, $k_2 = 7500$, then $R_k = 4$ and from Table 1, $\delta_o = 0.5392$ and $R_L = 1.3338$. The results are the same as in Example 1, but from equation (2) and (4), EL(0) and EL(δ_o) are 3 times their value in Example 1. In other words, if k_1 increases or decreases proportionally with k_2, then δ_o and R_L will not change. Also, EL(δ_o) is proportional to σ^2. If σ^2 is reduced through quality improvement, the process mean should be moved closer to the target value.

Sensitivity Analysis

The sensitivity of EL(δ_o) is measured by the relative increase in the EQL, RIL(δ), which is defined by the following equation:

$$RIL(\delta) = [\ EL(\delta)/EL(\delta_o) - 1] * 100\% \quad (5)$$

Table 2 shows that values of RIL(δ) for R_k = 2, 4, 6, 8 and 10. In this Table, column (A) represents the percent error in the actual δ relative to the optimal value, δ_o. Columns (B) through (F) of Table 2 are RIL(δ) for R_k=2,4,6,8 and 10 respectively. From this Table, if R_k=2 and $\delta=0.5\delta_o$, then RIL(δ)=1.9292%., i.e. a -50% error in δ_o causes only 1.93% increase in EQL, but if R_k=10 and $\delta=0.5\delta_o$, then RIL(δ) = 23.34%., i.e. -50% error in δ_o, cause 23.34% increases in RIL(δ). Further, as long as the error in δ is within ±10%, the increase in EQL is at most 1%. Therefore, the EQL is not very sensitive to the error in δ_o.

Conclusion

The above model showed that the process mean should shift slightly from the target value in order to minimize expected quality loss. Target selection of the process mean was formulated and computed. The model assumed that process variation would remain in control when the process mean changed. Some other models should be considered in future research, including: (1) unbalanced tolerance design with linear loss function; (2) unbalanced tolerance design with asymmetrical truncated quadratic loss function; and (3) unbalanced tolerance design with asymmetrical truncated linear loss function.

Acknowledgement

This research was supported in part by the National Science Council of the Republic of China, Grant No. NSC 89-2213-E-035-059-.

References

Bisgaard, S., Hunter, W. G. & Pallesen, L. (1984, February). Economic selection of quality of manufactured product. Technometrics, *26*, 9-18.

Boothroyd, G., Dewhurst, P., & Knight, W. (1994) Product Design for Manufacture and Assembly, New Yorks, Marcel Dekker.

Golhar, D.Y. (1987, April). Determination of the best mean contents for a canning problem, Journal of Quality Technology, *19*(2), 82-84.

Golhar, D.Y. & Pollock, S.M. (1988, July). Determination of the optimal process mean and the upper limit for a canning problem. Journal of Quality Technology, *20*(3), 188-192.

Gupta T. & Golhar, D. Y. (1991, January). Determination of optimal lot sizing parameters and a controllable process mean for a production system. International Journal of Production Research, *29*(4), 821-834.

Table 2: Sensitivity analysis of a quadratic loss function versus δ, relative increase in EQL (RIL(δ))

(A)	(B)	(C)	(D)	(E)	(F)
Percent Error	R_k				
	2.0	4.0	6.0	8.0	10.0
-50%	1.9292	7.9314	13.5788	18.6816	23.3374
-25%	0.4792	1.9330	3.2536	4.4112	5.4401
-10%	0.0764	0.3046	0.5076	0.6832	0.8351
-5%	0.0191	0.0758	0.1258	0.1687	0.2059
-1%	0.0008	0.0030	0.0050	0.0067	0.0081
1%	0.0008	0.0030	0.0050	0.0067	0.0081
10%	0.0760	0.2986	0.4910	0.6526	0.7911
25%	0.4732	1.8384	2.9943	3.9478	4.7526
50%	1.8809	7.1756	11.5086	14.9826	17.8451

Hunter, W. G. & Kartha, C. P. (1977, October). Determining the most profitable target value for production process. Journal of Quality Technology, *9*, 176-181.

Jeang, A. (1998, November). Tolerance optimization for quality and cost. International Journal of Production Research, *36*(11), 2969-2983.

Jeang, A. (1999, June). Robust tolerance design by response surface methodology. International Journal of Advanced Manufacturing Technology, *15*, 399-403.

Liggett, J.V. (1993). Dimensional variation management handbook. Englewood Cliffs, New Jersey, Prentice Hall.

Parkinson, P., Sorensen, C., & Pourhassan, N. (1993, March). A general approach for robust optimal design. Journal of Mechanical Design, *115*, 74-80.

Pfeifer, P. E. (1999, July). A general piecewise linear canning problem model. Journal of Quality Technology, *31*(3), 326-337.

Phadke, M. S. (1989). Quality Engineering Using Robust Design. Englewood Cliffs, New Jersey: Prentice Hall.

Taguchi, G., Elsayed, E.A. & Hsiang, T. (1989). Quality engineering in production systems. New York: McGraw-Hill.

Taguchi, G. (1991). Taguchi methods, research, and development, *Vol. 1*. Dearborn, MI: American Suppliers Institute Press.

Wu, C. C. & Tang, G. R. (1998, September). Tolerance design for products with asymmetric quality losses. International Journal of Production Research, *36*(9), 2529-2541.

Notation

The notations in this paper are defined as follows.

y: The quality characteristic.

μ: The process mean of a quality characteristic.

σ: The standard deviation of a quality characteristic.

$L(y)$: Quality loss function of a quality characteristic, y.

T: Target value of a quality characteristic.

Δ: The one side tolerance of a symmetrical specification.

A: The quality loss at the specification limits of a symmetrical quality loss function.

k: The coefficient of a symmetrical quality loss function.

k_1: The coefficient of an asymmetrical quality loss function on the left-hand side.

k_2: The coefficient of an asymmetrical quality loss function on the right-hand side.

Δ_1: The tolerance on the left-hand side of an unbalanced tolerance.

Δ_2: The tolerance on the right-hand side of an unbalanced tolerance.

$E[L(y)]$: The expected quality loss as a function of y.

R_k : The asymmetrical ratio of the coefficient of the quality loss function. i.e., $R_k = k_1/k_2$.

δ: The standardized distance of the process mean apart from target value. i.e., $\delta=(\mu-T)/\sigma$.

μ_o: The optimal process mean that minimizes expected quality loss.

δ_o: The optimal standardized distance of the process mean apart from target value. i.e., $\delta_o=(\mu_o-T)/\sigma$.

$EL(\delta)$: The expected quality loss as a function of δ.

$EL(0)$: The expected quality loss that the process mean is set at target value.

$EL(\delta_o)$: The expected quality loss that the process mean is set at the optimal value which minimizes expected quality loss.

R_L: The quality loss ratio. i.e., $R_L = EL(0)/EL(\delta_o)$.